

# The IFE Time Vector: An Analysis of Continuity and Change to Explain the Theories of Special Relativity and Quantum Mechanics

Daniel Brown<sup>1</sup>

*University College London, U.K.*

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## Abstract

This paper sets out to explain:

1. Why the speed of light  $c$  is a constant and is the maximum speed at which any moving entity can travel.
2. Why time elapsed differs for a moving entity relative to a stationary entity.
3. Reasons for the confusion between the wave and particle nature of an entity.
4. The relation between speed of light  $c$ , Planck's constant  $k$  and time.
5. The probability for a freely moving entity interacting in a particular spatial position.
6. Expressions for Mass and Momentum using this notation.
7. The minimum locus of uncertainty in position and momentum.

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<sup>1</sup>email: daniel.brown@youmeus.com

# 1 Identity

The conceptual apparatus for investigating time, space and energy and the concept of identity require revision when analysing change at the microscopic level.

We make use of the notion of change all the time. A cup of tea falls on the floor and it breaks. A broken cup results. A child transforms over time into an adult. A train moves along its track. A lump of metal is beaten into a ring.

Yet change is not a simple concept. It integrates interacting phenomena and concepts whose relations become progressively more difficult to define at increasingly fine levels of granularity.

Analysis of change requires a very precise reference of the identity of the thing that is changing. Paradoxically the net which precisely captures this concept must cover both that which was, together with that which is somehow different. Lao-tzu highlighted the difficulty with his observation: “if you realise that all things change, there is nothing you will try to hold onto”.

We require elucidation both of notions of continuity and of identity. Understanding what it means for a thing to remain the same clarifies what it means for that thing to change.

Logically we start with a theoretical examination of the smallest element of invariance possible - how a thing remains the same from moment to moment. Consider the smallest possible theoretically adjacent moments (the concept of adjacency is significant in understanding change) of the smallest possible thing.

To illustrate an example in everyday parlance, we can assume that when a teacup is hurled towards the floor, then at the points when it is flying through the air, it remains the same teacup. Once it lands and splinters to pieces on the floor, then we say that it changes into shards of pottery. If glued back together, then we describe it as “pretty much” the same teacup as at the outset.

We might, however, be more cautious in observation. Firstly, when the teacup accelerated and changed its speed (i.e. when launched and hurled) we could (pedantically) say that the teacup at that point changed from one entity into another. - Of course, we typically assert that the teacup did not *fundamentally* change, but that it was acted upon in such a way that it remained the same *thing* and that its *properties* changed - this is how we usually utilise the concept of identity to enable a definition of state change. However, at the microscopic level, it may not be possible to retain this separated concept of identity and associated properties.

We shall therefore include the concepts of space and even time in our most theoretically precise definition of “thingness” and proceed to examine

more closely the concept of invariance.

Consider a hypothetical stationary teacup (a theoretical unit of identity teacup that has no set of jiggling sub-components etc.). We observe this teacup closely from one moment ( $t_1$ ) to the next ( $t_2$ ) and are absolutely satisfied that this teacup has not changed in any way. - Specifically, we require that there has been a change in the time of the viewer (which implies that there has been a change in state of the viewer) but does not imply that there has been a change in time experienced by the teacup. Indeed, if nothing at all has changed in the teacup then we assert that time will not have progressed from the point of view of the teacup, and this, we assert, defines our most stringent notion of invariance.

We *define* the teacup as the unmoving matter at place  $x_1$  and at moment  $t_1$ . We can use a Space, Energy, Time (S,E,T) notation to describe the teacup:  $(x_1, e_1, t_1)$ .

We make the following Assumptions:

- (i) time, space and energy advance in quantised units
- (ii) time can only advance when change occurs
- (iii) change can only occur if there is either or both:
  - (a) change in energy
  - (b) change in spatial position

If an entity changes spatial position or changes energy, then we assume that each of these changes takes a quantity of time which we shall define as  $t^*$  and  $t'$  respectively (later we shall see that these components can be combined as vectors on two orthogonal axes and it is the purpose of this paper to explain the very great significance of the combination of these two elements for the advance of time). Then we have  $(x_2, e_1, t_2^*)$  or  $(x_1, e_2, t_2')$  (here, as elsewhere we shall use  $t^*$  and  $t'$  to refer to the smallest possible units of change in space-time and energy-time respectively) and we typically explain that it is the matter but not its position or the time associated with that matter that constitutes its identity. Indeed, we take this so much for granted that the last clause of this phrase sounds very awkward. However, at the microscopic level, these features cannot be considered as we might traditionally expect.

At the microscopic level, we shall consider an entity through all three elements of its matter, its position in space, and its position in time. We can then explain (partial) continuity of identity in different ways - for example: (i) continuity of energy over a changing arena of space and time. (ii) continuity of space over a changing arena of energy and time

This complexity in identity definition is highlighted in the macroscopic world in the paradox of the ship of Theseus where over a period of time, in order to repair a wooden ship - which we can refer to as Ship 1 - its planks are replaced one by one. However, the original planks are taken aside and re-

constituted in identical architecture into another ship at a different location which we call Ship 2. The question of identity is which ship is the original ship of Theseus. - In order to decide, we need to decide on our criterion of identity: either it is continuity of matter over changing space and time (Ship 2) or continuity of space over changing matter and time (Ship 1). In a very real sense, both ships represent two parallel continuities of identity over changing time. This forms a useful analogy for this paper as it relates to the potential for an object to be conceived as in two alternative positions at the same time.

How we regard the identity of an entity can therefore affect both what and where we presume that entity to be. In particular, it is possible to view an entity as being at two different points in space at the same time, dependent on how we have tracked and how we collapse its identity.

The rest of this paper will be concerned with the different implications for a change in time caused by a change in spatial position and a change in time caused by a change in energy, and the consequences that this dual (vector) approach to time has in terms of relativity and quantum mechanics.

## **2 Interrelating Fluctuating Entities**

In order to analyse more precisely the mechanics of a vector approach to time, it is necessary to formalise a set of rules governing the interaction of time, space and matter. A simple example in the macroscopic world is what

has recently been termed the “Mexican wave” in football stadiums. This illustration is important philosophically because we can examine the contribution of each individual fan and visualise subtle interactions at a theoretical level.

We shall explore the mechanism by which fans propagate a wave to undulate across the stadium. - We assume first of all that the stadium has only two rows A and B. Row A comprises  $n$  adjacent fans. In row B we are only concerned with two fans: one at the start of the row and the second at the  $n$ th position. Each fan can move through a cycle of discrete states of standing up and sitting down as illustrated in diagram 2.<sup>2</sup>:

DIAGRAM 1

Row A      △△△△△△△△△△△△△△ → moving fan event

Row B             $\triangle$  fan1                                   $\triangle$  fan n

The two fans in Row B measure time elapsed whilst remaining stationary (these are simply individual fans bouncing up and down and not interacting with other fans) and counting energy changes. Time elapsed is also measured by the “fan event” in Row A travelling from the position of fan 1 to the posi-

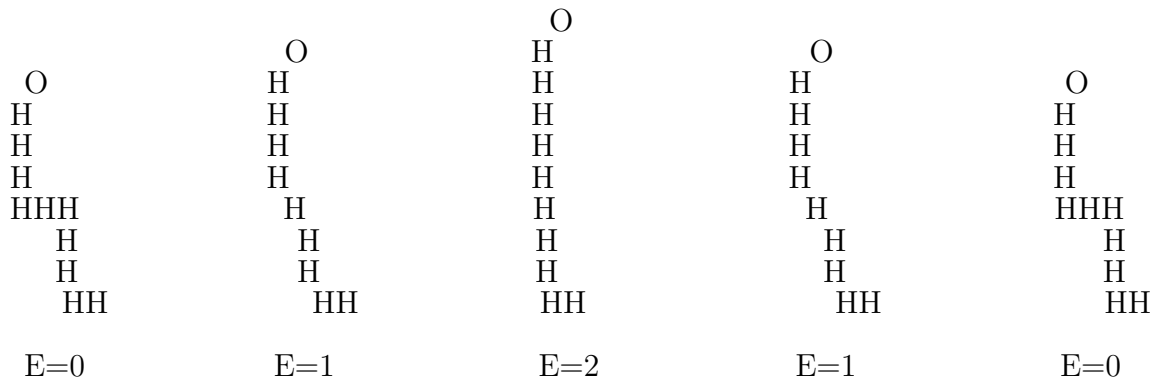
<sup>2</sup>the lateral effects of entities on each other are significant; however in this paper we shall examine just one spatial dimension.

tion of fan n. We shall examine carefully how each fan measures passing time.

The following rules enable us to specify - and vary - the propagation of a wave around the stadium:

## DIAGRAM 2

(Different Energy states represented by a fan standing up and sitting down)



1. All fans move in exactly the same way
2. Each fan has a certain number of discrete states  $q$ . We will call these “energy states” e.g. in Diagram 2 there are 3 discrete energy states 0,1 or 2. Each fan is capable of registering states of energy, a quality which we annotate by  $e$ . We indicate the first energy position by 1, the next by 2...100 etc.
3. A fan can have an effect on an adjacent fan only.
4. A fan is activated when its adjacent neighbour moves from one speci-

fied energy state to another - the “trigger point”  $p$ . For example when a fan moves from  $1 \rightarrow 2$ . This trigger point can be varied. We could alternatively specify that the downward motion of moving from  $2 \rightarrow 1$  initiates an upward motion of  $0 \rightarrow 1$  in an adjacent fan. A fan will not commence moving until its adjacent fan passes through the trigger point.

5. Once a fan is acted upon, this fan will move through a complete cycle of energy states (e.g. Energy moves through 0,1,2,1,0 in the diagram above).

6. Each fan can *itself measure time only through a change in energy-state*. Each fan records time by holding up a card which indicates its registered time for which 1 indicates the first unit of time, 2 the next ...100 etc. Thus the amount of time recorded by a fan to reach its trigger point at any spatial position is  $pt'$ .

7. Each change of energy state causes a change  $t'$  in local time where local time applies to the current fan.

8. Each fan sits a distance  $d$  apart from each other fan. A spatial variable  $S$  indicates the position of each fan: 1 indicates the position of the first fan, 2 that of the next fan ...100 the  $100^{th}$  fan etc. Hence if we move from the first fan to the  $n$ th fan, the distance travelled is  $n(-1)d$ . For convenience we can refer to a fan at the 0th position and hence distance travelled is  $nd$ .

9. Each activation of a neighbour is instantaneously associated with a change in non local time  $t^*$  for adjacent fans (but not the moving fan itself) - this is effectively the time taken for the energy movement to be propagated from one location to its adjacent position. Significantly it appears empirically that generally  $t^* > t'$  and that  $pt' \gg t^*$

10. The unit change in time  $t^*$  caused by a change in space is distinct from the unit change in time  $t'$  caused by the change in energy.

11. The final position of a fan is determined by an *interaction*. This occurs at a specific spatial position and energy. Once an interaction has occurred, then all activities of that fan event cease at that time.

There cannot logically be a change in time without a change in either energy or space. Causally, therefore, for a given entity in a specific fixed spatial position, then with no change in energy *there can be no change in time*.

We now have a method for indicating the position of state of each fan at each position according to (S,E,T) which defines a position in space, a state of energy and a position in time. The combination of rules with the method of representation enables us to abstract from the fans themselves to examine the consequent dynamic interacting fluctuating entities that arise.

We indicate a change of energy by (e), a change of spatial position by (s) and a change in time by (t). Likewise **e**, **s**, and **t** indicate that these quantities of energy, space and time respectively remain the same. Since they are in a cycle of interaction, it is not appropriate to attribute the concept of “thingness” to time, space or energy in isolation and we shall therefore refer to these virtual concepts as entities only.

We can then extract a set of basic Interrelated Fluctuating Entity - IFE - Rules (“iffy” conveniently catches the component temporal vagueness) which determine a movement of the disturbance through space, energy and time:

### **IFE Causal Rules**

(i) A change in energy (e) at a certain position in space and time causes an (instantaneous) increment in time ( $t'$ ) associated with the new energy state at the same position in space **s**:

**(e)  $s \rightarrow (t')$  a change in energy over constant space causes a change in time**

(ii) A change in time at a certain position in space associated with a certain energy causes a change in energy (e) at the same position in space **s** - at the later time. Rules (i) and (ii) will continue in oscillation at the same space for  $p$  oscillations until the trigger point is reached. It is useful to consider a simple instance for a single movement in space where  $p=1$ , i.e. where a single movement in energy is sufficient to result in a movement in

space.

(where  $p > 1$  then we can consider the cycling of oscillations between increments in time and energy as taking  $(p-1)$   $t'$  following which the energy reaches the trigger point so that there is effectively a movement in space once there have been  $p$  energy increments).

(iii) If the change in energy ( $e$ ) at a certain position in space and time passes the trigger point  $p$ , then this causes an increment in space ( $s$ ) associated with the new energy state at the same position in time  $t$ :

**(e)  $t \rightarrow (s)$  a change in energy over constant time causes a change in space**

(iv) The change in space ( $s$ ) at a specific time of a specific energy causes an instantaneous increment in time ( $t^*$ ) associated with the energy state at the new position in space:

**(s)  $e \rightarrow (t^*)$  a change in space over constant energy causes a change in time**

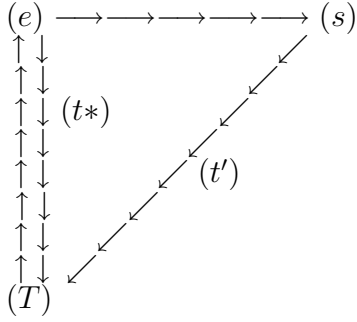
These IFE rules are sufficient to define a disturbance which moves with a constant velocity through space and time. It will be noted that the disturbance has inertia and will continue to move indefinitely with this constant velocity (until it interacts with another entity).

At the trigger point a change in ( $e$ ) results **both** in a change in energy-

time  $(rt')$  **and** - because the trigger point has been passed - a change in space  $(s)$ . The change in  $(s)$  causes a change in space-time  $(nt^*)$ . The total magnitude of time elapsed must then comprise both  $(rt')$  and  $(nt^*)$ .

We can (crudely) illustrate in two dimensions the causal cycle:

DIAGRAM 3



We note some important consequences. Firstly, it is the occurrence - at the trigger point - of both a movement in space and its associated movement in time, and the movement in time associated with this change in energy which results in a fundamental ambiguity for a given magnitude of time - where an entity is located in space and what its energy is. Secondly it is the causal link between a movement in energy, a movement in space and an associated increase in time which is responsible for the maximum speed of light. Thirdly there is a possibility for a very small single movement in energy and space without an associated movement in time which has potential implications for the collapse of an IFE which would in principle permit particular effects over a distance (produced specifically by a “domino”

effect of single movement rippling across adjacent spatial positions) at a speed greater than light.

The consequence is that from a temporal perspective there has been both an advance in energy-time  $t'$  and an advance in spatial-time  $t^*$ . We must then resolve these separate but coterminous advances in time which commence from the same initial moment into a single magnitude.

Analysing these dual advances in time is not at first natural to our way of thinking. We have to consider that **as the changes in time  $t'$  and  $t^*$  occur, they operate from the same start point instantaneously: the increments apply in both instances to  $t_1$  and whilst  $t^*$  is logically subsequent to  $t'$ , it is not temporally subsequent:  $t^*$  and  $t'$  therefore apply in unison from the same moment.**

An alternative way of viewing this is consider that time advances in two ways at once: on the one hand through a movement in energy and on the other through a movement in space. And both occur coterminously.

We define the total time state or time magnitude of an entity at a certain spatial position and energy to include both the time  $t'$  taken for changes in energy position and the time  $t^*$  taken for changes in spatial position. “Experienced” time, which depends solely on  $t'$ , may vary across a distance between a moving entity and a stationary one, but **all interactions occur between entities which are in the same time state.**

Let us examine the time state for adjacent observers and a moving entity:

DIAGRAM 4

(Row A)  $\triangle\triangle\triangle\triangle \rightarrow$  moving fan

(Row B)  $\triangle_1\triangle_2$  (1,2) indicates position number

We can compare the time *experienced* by the spatially moving entity for it to move in Row A from adjacent point 1 to point 2, and the *time state* of this spatially moving entity when it reaches position 2.

For the spatially moving entity, the time *experienced* is simply  $pt'$ . However, the time state of this entity, which corresponds with the time *at which an interaction can occur* will be different.

The complete time state of the moving entity which will be observed by the adjacent fans in row B must include the time for spatial movement (which will not have been *experienced* by the fan)  $t^*$ .

From Diagram 3, we note that:

(i) the increase in time  $t'$  which is caused by the change in energy applies from the initial time  $t_1$

(ii) the increase in time  $t^*$  which is caused by the change in space also

applies from the initial time  $t_1$

**Both increases in time  $t'$  and  $t^*$  operate at once, at “the same time”.** In order to calculate the time state: we cannot simply add  $t'$  and  $t^*$  for these increments are operating simultaneously and originate from the same initial time  $t_1$ .

These advances in time operate in unison, and together they define the quantity of advance in time. **In order to combine these coterminous advances in time which proceed along different axes of energy-time  $t'$  and space-time  $t^*$  into a single time magnitude, we shall make the following hypothesis: that these axes are orthogonal and hence that their combination comprises a simple pythagorean sum.** It is remarkable that the consequence of these two orthogonal dimensions of time is both the theories of special relativity and quantum mechanics as will be shown.

Let us consider what occurs in a time of magnitude  $|T|$  from a start time  $t_1$

$$|T| = t_1 + \sqrt{(t^*)^2 + (t')^2}$$

Where the start time  $t_1 = 0$  then

$$|T| = \sqrt{(t^*)^2 + (t')^2}$$

In the case where the trigger point  $p$  is greater than 1 then:

$$|T| = \sqrt{(t^*)^2 + (pt')^2}$$

And for a sequence of n spatial movements:

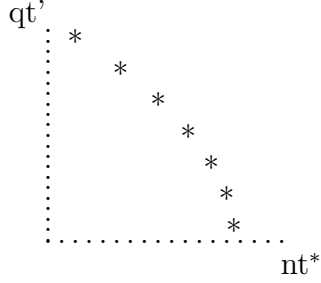
$$|T| = n\sqrt{(t^*)^2 + (pt')^2} = \sqrt{(nt^*)^2 + (npt')^2}$$

Following a series of n spatial movements, in the final nth spatial position there follows a sequence of variable r energy movements (which may exceed the trigger point i.e. we can have  $r > p$ ).  $rt'$  corresponds to the actual **detected** energy of the IFE at the point of its interaction where energy  $e = \frac{h}{rt'}$ . We will refer to  $\frac{1}{rt'}$  as the “frequency” of the IFE, since this mathematically corresponds with the terminology used in the literature. However, it will be seen that the  $rt'$  term is somewhat different conceptually. Since  $rt'$  is measured energy-time  $t'$  then we can form the complete expression for the time magnitude:

$$|T| = \sqrt{(nt^*)^2 + (npt' + rt')^2} \quad (1)$$

Since n and r are variables, there exists a range of alternative combinations of energy and spatial position states which combine to form the same time magnitude  $|T|$  from variable components of energy-time  $t'$  and space-time  $t^*$ . We can represent this for a fixed  $|T|$  of magnitude  $qt'$  as a “temporal arc” (see as in diagram 5 below):

Diagram 5 - temporal arc for a photon at a time  $|rt'|$



**All points on the temporal arc have the same time magnitude.**

It is notable that we can represent the time state as a complex vector. Thus using a notation with spatial-time ( $t^*$ ) as real and energy-time ( $t'$ ) as imaginary:

$$\mathbb{T} = nt^* + \imath(np + r)t' \quad (2)$$

or where  $z = (p + r/n)$  :

$$\mathbb{T} = n(t^* + \imath zt') \quad (3)$$

Differences in experienced time between moving and stationary entities all stem from the difficulty in measuring  $t^*$ .

### 3 Properties of Interrelating Fluctuating Entities

We can now define some properties of interrelating fluctuating entities:

1. the time to travel from one IFE to the next is a complex number (or equivalently a two dimensional vector) where

$$\underline{T} = nt^* + \imath(np + r)t'$$

or where  $z = (p + r/n) :$

$$\underline{T} = n(t^* + \imath zt')$$

$$\text{Time Magnitude } |\underline{T}| = (T) = \sqrt{T T^*} = \sqrt{(nt^*)^2 + (npt' + rt')^2}$$

For large  $n$ , we can often ignore the residual  $rt'$  energy-time component in calculations of time magnitude. For increasingly small distances, however, it becomes increasingly significant.

$$2. \text{ Frequency } f = \frac{1}{\imath(rt')}$$

3. The speed of propagation

$$v = \frac{nd}{|T|} = \frac{nd}{\sqrt{(nt^*)^2 + (npt' + rt')^2}} \quad (4)$$

$$4. \text{ Wavelength } \lambda = v/f = \imath qt'v$$

$$\lambda = \frac{\imath(nd)(qt')}{\sqrt{(nt^*)^2 + (npt' + rt')^2}} \quad (5)$$

5. A consequent maximum speed is implied for which a wave can theoretically propagate through the medium. This will occur when the trigger point  $p$  is zero. i.e.

$$v_{max} = \frac{d}{\sqrt{(t^*)^2 + (0)^2}} = \frac{d}{t^*} \quad (6)$$

This is significant as it represents purely the time taken to move spatial distances by an entity where no energy changes are occurring.  $v_{max}$  is the speed of light  $c$ , and the total absence of energy-time as a component in the time magnitude explains why such a speed cannot be exceeded and why for an entity travelling at such a speed, we would expect no time to be experienced by that entity at all.

## 4 The Magnitude of the Time Vector (over large distances)

From the Time vector equation:

$$T = n (t^* + i z t') \text{ where } z = (p + r/n)$$

We can calculate the magnitude of this time vector as:

$$|T| = n \sqrt{(t^*)^2 + (zt')^2} \quad (7)$$

In a sense this simple equation is all we need to express the theory of relativity, for  $|T|$  expresses the total time magnitude and  $(zt')$  represents the time experienced by the moving IFE. In order to demonstrate how this

accords with the familiar model of the theory of relativity, we can further calculate:

$$\text{Speed } v = \frac{nd}{n\sqrt{(t^*)^2 + (zt')^2}}$$

$$v = \frac{d}{\sqrt{(t^*)^2 + (zt')^2}} \quad (8)$$

we can assert also that for the speed of a photon over a significant distance there is no trigger point (i.e.  $p=0$ ) and  $q/n$  will be comparatively very small then:

$$\text{Speed } c = \frac{nd}{nt^*} = \frac{d}{t^*} \quad (9)$$

Rearranging (7):

$$|T| = n \left( \frac{(t^*)^2}{\sqrt{(t^*)^2 + (zt')^2}} + \frac{(zt')^2}{\sqrt{(t^*)^2 + (zt')^2}} \right)$$

Substituting from (8) and (9) into the first part of the expression and rearranging the second part:

$$|T| = \frac{nv(t^*)}{c} + n(t^*zt') \frac{(zt')}{t^*} \sqrt{\frac{1}{(t^*)^2 + (zt')^2}}$$

Further rearranging the second part:

$$|T| = \frac{nv(t^*)}{c} + n(t^*zt') \sqrt{\frac{(t^*)^2 + (zt')^2 - (t^*)^2}{(t^*)^2[(t^*)^2 + (zt')^2]}}$$

From which we obtain:

$$|T| = \frac{nv(t^*)}{c} + n(t^*zt')\sqrt{\frac{1}{(t^*)^2} - \frac{1}{(t^*)^2 + (zt')^2}} \quad (10)$$

But from (8) and (9) we have:

$$\frac{\sqrt{c^2 - v^2}}{c} = t^*\sqrt{\frac{1}{(t^*)^2} - \frac{1}{(t^*)^2 + (zt')^2}} \quad (11)$$

Substituting this expression into (10) we obtain:

$$|T| = \frac{nv(t^*)}{c} + n\frac{\sqrt{c^2 - v^2}}{c}(zt') \quad (12)$$

Now in terms of distance travelled  $x$ :

$$x = c(nt^*)$$

Substituting into (12) we arrive at:

$$|T| = n(zt')\sqrt{1 - v^2/c^2} + (v/c^2)x$$

Since  $n(zt')$  corresponds to  $\tau$  the amount of time experienced from the perspective of the moving entity and  $|T|$  corresponds to the time observed by a stationary observer, this is the familiar Einstein-Lorentz expression:

$$\tau = \gamma(|T| - (vx/c^2)) \text{ where } \gamma = (1 - v^2/c^2)^{-1/2} \quad (13)$$

The simplicity and explanatory power of the vector approach is notable by comparison. Even in cases (such as in the calculation of combined velocities later in this paper) where there may be an eruption of terms, there is a

fundamentally comprehensible approach, which is not always the case with Einstein-Lorentz presentations.

It is important to recognise that all our understanding of “relativistic” effects are fundamentally underpinned by time and time alone. Calculations for alterations in distance arise from the perception of measured space through velocities which ultimately refer back to differences in experienced time derived from the difference between combination of space-time and energy-time.

## 5 Planck’s constant and the speed of light $c$

It is mathematically trivial but philosophically interesting to examine the constants that underlie the units of the axes of space-time  $t^*$  - in terms of distance, and energy-time  $t'$  - in terms of energy.

We know that Distance = Speed . Time:

Then

$$x = nct^* \quad \text{i.e. } \Delta x = ct^* \quad (14)$$

Where  $c$ , the speed of light is revealed as the constant that connects the smallest possible quantum spatial distance  $d = \Delta x$  to the smallest quantum of spatial-time  $t^*$ .

It has been observed empirically that Energy =  $h$ . Frequency:

$$e = \frac{h}{\iota(rt')} \quad \text{i.e. } \Delta e = \frac{h}{\iota(t')} \quad (15)$$

Where  $h$ , Planck’s constant is revealed as the constant that connects the

smallest possible quantum in energy  $\Delta e$  to the smallest quantum of energy-time  $t'$ .

This refines our view on the meaning of the constants  $c$  and  $h$  and the connections from time to distance and energy respectively.

## 6 The probability for a freely moving entity interacting in a particular spatial position

The magnitude of the time vector associated with the notion of the temporal arc indicates that at a specified time magnitude, there are variable combinations of spatial position and energy which can combine in an IFE to form this same time magnitude. For small distances, the contribution of the energy-time component  $irt'$  becomes increasingly significant and when the total time magnitude  $|T|$  measured is of the order of  $|irt'|$  then there will be challenges in precisely divining specific energy and spatial position. Indeed it is immediately evident that for a given  $|T|$  there is unavoidably some uncertainty in defining these qualities, and that this uncertainty will become increasingly pronounced for smaller values of  $|T|$ . **It is because there are alternative compositions of spatial-time (nt\*) and energy-time (rt') for a given time magnitude  $|T|$  that only a statistical method can be used to define the position in space and energy of the IFE.**

Calculation of  $P(x)$  the probability of the IFE being located (through an interaction) in a specific position is somewhat more intricate than might at

first be expected.

Consider an IFE starting from an initial time  $|T| = 0$ . We might assume that the particle has an interaction at spatial position  $x$ . Then we might expect that having arrived at position  $x$  we must consider each of the temporally precedent spatial positions where an interaction did NOT occur: NOT( $x-1$ ), NOT( $x-2$ )... where there could have been but was no interaction. For a probability distribution that was identically and uniformly distributed this would be straightforward - we could examine the  $n$  positions - each separated by a distance  $\Delta x = d$  prior to the interaction at  $x$ :

$$\begin{array}{c} \text{.....}^* \\ x = n\Delta x \end{array}$$

We can define the probability of occurrence in a very short space  $\Delta x$  as  $(B\Delta x)$  where  $B$  is the probability density.

So the probability of non-occurrence at a very short space is  $(1 - B\Delta x)$

If a distance  $x$  is travelled before an interaction then where  $P(x)$  is the probability density:

$$P(x)\Delta x = (1 - B\Delta x)^n B\Delta x$$

$$P(x) = B(1 - B\Delta x)^n$$

For a large  $x$  then  $n = \frac{x}{\Delta x} \longrightarrow \infty$

i.e. we might at first expect:

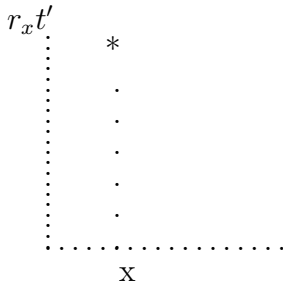
$$P(x) = Be^{-Bx} \tag{16}$$

However,  $B$ , the probability density of an interaction at each position varies according to the number of alternative energy positions at each possible spatial position  $x$ . As we look more carefully at the energy position alternatives at each spatial position  $x$  we see that the range of possible *energy positions itself* will vary at different spatial positions. We therefore have to examine the probability density of position in a very short space as a variable which depends on energy levels and which we will label  $B(rt')$ . We are using the probability of energy position as the probability density of spatial position at a small point in space.

Let us assume that for each occasion that the IFE moves from one energy position to another or from one spatial position to another there is a primary uniform probability  $A$  of interaction for an IFE with another (group of) IFEs (that depends on the state of the other group of IFEs).

We might therefore assume that to arrive at the probability for an interaction at a specific energy position ( $rt'$ ) at a spatial position  $x$  we sum all of the probabilities for each possible energy position at  $x$  (see Diagram 6 below).

Diagram 6



Consider the probability  $P(r_x t')$  for an interaction at a single energy position  $(rt')$  at spatial position  $x$ . We note that in order for there to be an interaction at the energy position  $(rt')$ , we must have had no interactions at each of the previous possible (and temporally precedent) energy points  $(r-1)t'$ ,  $(r-2)t'$  etc.

To calculate the probability of an interaction at a particular energy position we use a similar method to that initially assumed for spatial position.

We define that the primary uniform probability of an interaction  $= A$

Now for a particular spatial position  $x$  the probability density of having an interaction at one of the energy positions will be inversely proportional to the time taken to move through the potential number of energy positions at  $x = A \cdot \frac{1}{r_x t'}$  where  $r_x$  is the number of possible energy positions at  $x$ .

i.e. the probability of interaction in a short interval of time  $= A \frac{1}{r_x t'} t'$

And the probability of non-occurrence in a short interval of time is  $(1 - A \frac{1}{(r_x t')} t')$

If an interval of time  $rt'$  passes before an interaction then where  $P(rt'/x)$  is the probability *density* of  $(rt')$  at a given  $x$ :

$$P(rt'/x) = A(1 - A \frac{1}{(r_x t')} t')^r = A(1 - A \frac{r(t')}{r(r_x t')})^r$$

For large  $(rt')$  then  $(1 - A \frac{rt'}{r(r_x t')})^r$  tends to  $e^{-A \frac{rt'}{r_x t'}}$

$$P(rt'/x) = A e^{-A \frac{(rt')}{r_x t'}} \quad (17)$$

It is straightforward to calculate mean and variance using this.<sup>3</sup>

However, we must consider not only a single given spatial position at

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<sup>3</sup>note that we can calculate the mean and variance as follows:

x (=nd), but further alternative possible *spatial* positions such as at x = (n-1)d, (n-2)d...etc.

Since the total time magnitude  $|T|$  can be composed in more than one way, then for a particular energy position  $(rt')$  we must consequently consider not only non-occurrences at  $(r-1)t'$ ,  $(r-2)t'$ ..., but also for each of these energy positions, the non occurrences at all the coterminous *spatial* positions which provide the same time magnitude  $|T| = |rt'|$ . In this case, in order to establish  $P(|T|)$  we must consider all the ways in which it can be formed from the combination of the the first spatial position, the second spatial position etc... It is sometimes useful to highlight  $P(|rt'|) = B(rt')$  when it can be over more than one spatial position.

Consider firstly a second possible spatial position only. Given a *specific* spatial position then calculation of the probability of an energy position  $(rt')$  requires us only to consider all the possible alternative energy positions where no interaction occurred at  $(r-1)t'$ ,  $(r-2)t'$ ...etc at that given spatial position.

$$Mean = k_o = \langle k \rangle = \int_{-\infty}^{\infty} k(A)e^{-kA}dx = \frac{A}{A^2} = \frac{1}{A}$$

$$variance = \sigma_k^2 = \langle k^2 \rangle - (\langle k \rangle)^2$$

$$\langle k^2 \rangle = \int_{-\infty}^{\infty} k^2(A)e^{-kA}dx = \frac{2A}{A^3} = \frac{2}{A^2}$$

$$i.e. \sigma_k^2 = \frac{1}{A^2}$$

However, with an alternative possible spatial position we must account also for all the possible energy positions at the second spatial position which in combination with the spatial-time  $t^*$  (caused by the movement in spatial position) can comprise the same time magnitude equal to  $|rt'|$  in the first spatial position. These possible positions for a certain energy state ( $rt'$ ) at a specific position  $x$  (=nd) potentially exist only for those combinations of spatial and energy positions which have the same time magnitude  $|T| = |rt'|$  such that  $|T| = \sqrt{(nt^*)^2 + (npt' + rt')^2}$  where  $r$  is the energy position that can occur at any spatial position.

This requires us to account for the probability density of the potential spatial position at  $x$  which itself accounts for the permitted probability density  $P(r_x t')$ . This probability for a potential spatial position  $r_x$  is simply:

$$P(r_x) = \frac{1}{(nd)}d \quad (18)$$

Let us consider the temporal arc in more detail. We can see that for each possible interaction at a specific spatial position  $x$  and energy position ( $rt'$ ) we must consider all of the possible interactions at energy and spatial positions on an arc associated with it. We should consider firstly how many possible positions are on this arc. Here we make use of a calculation originated by Gauss for analysing a fundamental point lattice (see below).<sup>4</sup>

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<sup>4</sup>We essentially wish to know the number of *potential* positions on the temporal arc formed through the time magnitude  $|T| = \sqrt{(nt^*)^2 + (npt' + rt')^2}$ . Since  $t^*$  and  $t'$  are finite numbers, and since  $n$ ,  $p$  and  $r$  are integers then there exist only a small subset of positions on the temporal arc that can exist to form  $|T|$ . Since this can effectively be

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represented as the root of a sum of two squares, then we effectively want to estimate the number of lattice points  $C(|T|)$  on the circumference of a circle of radius  $|T|$ .

We can apply a theory of point lattices for determining the number of possible lattice points *in and on* a circle  $C(|T|)$  of radius  $|T|$ . If we consider the circle at the origin of a fundamental point lattice with each lattice point as the centre of a unit square with sides parallel to the axes  $t^*$  and  $t'$ , then we can analyse the area of all the squares whose *centres* are inside or on  $C(|T|)$ . This area  $L(|T|)$  will comprise a number of complete squares entirely within the circle, but also *a number of squares that are divided by the circle of radius  $|T|$*

Some parts of squares with *centres* inside the circle of radius  $|T|$  will remain outside of the circumference, and equally there are some squares with *centres* outside the circle whose boundaries fit partly within the circle's perimeter. If we theoretically shade in all the complete squares whose centres are in or on the circle, then we can bound the shaded area  $L(|T|)$  from below and above - we find the largest disk whose interior is completely shaded, and the smallest disk whose exterior is completely unshaded. Since the diagonal of a unit square is  $\sqrt{2}$  then all shaded squares must be contained in a circle of radius  $= |T| + (\sqrt{2}/2)$ . Similarly the circle whose radius  $= |T| - (\sqrt{2}/2)$  is contained entirely within the shaded squares.

Consequently

$$\pi(|T|^2 - \sqrt{2}|T| - \frac{1}{2}) \leq \pi(|T|^2 - \sqrt{2}|T| + \frac{1}{2}) \leq L(|T|) \leq \pi(|T|^2 + \sqrt{2}|T| + \frac{1}{2})$$

Which implies that

$$|\frac{L(|T|)}{|T|^2} - \pi| \leq \pi(\sqrt{\frac{2}{|T|^2}} + \frac{1}{2|T|^2})$$

Since  $(\sqrt{\frac{2}{|T|^2}} + \frac{1}{2|T|^2})$  tends to 0 as  $|T| \rightarrow \infty$  then  $L(|T|)/|T|^2 \rightarrow \pi$

i.e.  $L(|T|) = \pi|T|^2$ .

This defines the number of lattice points both in and on a circle of radius  $|T|$ . We require the number of points solely *on* the circle of radius  $|T|$ . Using elementary geometry this is simply  $C(|T|) = 2\pi|T|$ .

Whilst the behaviour will be irregular in that different arcs will have volatile numbers

This shows that  $C(|T|)$  the number of permissable points on a temporal arc that can compose a time magnitude  $|T|$  is:

$$C(|T|) = 2\pi|T| \quad (19)$$

If we examine the temporal arc closely, we see that to calculate the probability of a particular energy ( $rt'$ ) we need to account not only for all of the potential interactions that did *not* occur at energy positions  $(r-1)t'$ ,  $(r-2)t'$ ... but also for all of the feasible interactions that could have, but did not occur at energy positions such as  $(|rt' - t^*|)$ ,  $(|rt' - t^* - 1|)$ ... at a second *spatial* position - and further  $(|rt' - nt^*|)$ ,  $(|rt' - nt^* - 1|)$ ... at the  $n$ th spatial position.

***Calculation of the probability of NON-interactions - represented by NOT (...) - requires us to sum the area of the arc of every possible position at every possible spatial position***

Diagram 7

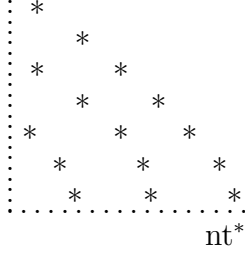
qt'

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of potential compositions through  $nt^*$  and  $rt'$  (and some arcs will be effectively prime, composed through only a single instance of  $n$  and  $rt'$ ) we can operate with an average value for the number of possible positions on a variable temporal arc, which will be effective if summed over a large/infinite series - which is how we will be performing our summation of probabilities. We therefore sum the first  $n$  values of  $L(|T|)$  (the number of possible lattice positions on a circle of radius  $|T|$ ) and divide by  $n$  to obtain an associated average for the total number of ways for combining the two axes of time to form the single time magnitude:

$$\frac{C(|T|)}{|T|} = \frac{C(0)+C(1)+C(2)+\dots+C(|T|)}{|T|}$$

We can therefore utilise  $C(|T|) = 2\pi|T|$ .



The mechanics for this calculation are facilitated if we work backwards and investigate historically the non-occurrences of interactions for spatial and energy positions.

To illustrate this technique, let us consider a simplified example. For this example, we shall imagine the probability of NOT having an interaction at an energy magnitude of 3 where there are only 3 possible spatial positions and 3 possible energy positions. In order to (over)simplify this example further, we shall also assume that each spatial position involves a straightforward addition of a single value  $t^* = t' = 1$ .

Then we arrive at a layered iteration of probabilities that form a NON-interaction :

$$\begin{aligned}
P(\text{NOT}(|(rt')|3)) &= \text{NOT}[(x_03)(x_10)(x_20) + (x_00)(x_12)(x_20) + (x_00)(x_10)(x_21) + (x_01)(x_11)(x_20)] \\
&\quad \vdots \\
&\quad \text{NOT}(|(rt')|2) \dots \quad \text{NOT}(|(rt')|1) \\
P(\text{NOT}(|(rt')|2)) &= \text{NOT}[(x_02)(x_10)(x_20) + (x_00)(x_11)(x_20)] \\
&\quad \vdots \\
&\quad \text{NOT}(|(rt')|1) \\
P(\text{NOT}(|(rt')|1)) &= \text{NOT}[(x_01)(x_10)(x_20)]
\end{aligned}$$

It is not surprising that matrices have been applied in quantum mechanics; however we shall not analyse this method further in this paper.

There are two important points to note here. Firstly, because we work backwards, we are investigating non-occurrences of interactions and this means that instances such as  $(x_01)(x_11)(x_20)$  must be considered probabilistically - even though there is no such single possibility - i.e. the first  $(x_01)$  is an instance of something that did not occur *in the history* of the second  $(x_10)$ .

Secondly, for the  $\text{NOT}|(rt')|3)$  we include all four possible groupings as well as all the  $\text{NOT}|(rt')|2)$  non-happenings (and all of the  $\text{NOT}|(rt')|1)$  in turn).

In order to simplify our calculation, we shall first make use of a symbol  $k$  to combine from equations 17,18 and 19:

$$k = \frac{2\pi|T|}{(n)(rt')}$$

It will be noticed, interestingly, that  $k = \frac{2\pi}{\lambda}$

We note that through the symmetrical character of the squared time magnitude there will be the same number of *available* energy and spatial positions.

We can use an efficient summation method which enables us to aggregate all the possible probabilities. To illustrate this, we can first calculate notionally for two spatial positions only - (i.e. provided that there are only 2 energy positions)

$$P(rt') = B(rt') = B(k) = \int_{-\infty}^{\infty} Ae^{-Ak'} . Ae^{A(k-k')} dk'$$

With the constraint that  $k'$  and  $(k-k')$  are not negative - i.e. both  $Ae^{-Ak'}$

and  $Ae^{A(k-k')}$  are effectively Heaviside step functions which we can represent with the addition of  $H(k')$  and  $H(k-k')$ :

$$\begin{aligned}
& \int_{-\infty}^{\infty} Ae^{-Ak'} H(k') . Ae^{A(k-k')} H(k-k') dk' \\
&= \int_0^k Ae^{-Ak'} . Ae^{A(k-k')} dk' \\
&= A^2 k e^{-Ak}
\end{aligned}$$

Similarly for 3 positions we have :

$$\begin{aligned}
& \int_{-\infty}^{\infty} A^2 k e^{-Ak} H(k') . Ae^{A(k-k')} H(k-k') dk' \\
&= \int_0^k A^2 k e^{-Ak'} . Ae^{A(k-k')} dk' \\
&= \frac{A^3}{2} k^2 e^{-Ak}
\end{aligned}$$

and for all the possible  $n$  positions across the temporal arc we can see through inference that we obtain:

$$P(k) = \frac{A^n k^{n-1}}{(n-1)!} e^{-Ak} \quad (20)$$

Note that:

$$Mean = k_o = \langle k \rangle = \int_{-\infty}^{\infty} \frac{(kA)^n}{(n-1)!} e^{-kA} dx = \frac{n}{A} \quad (21)$$

$$variance = \sigma_k^2 = \frac{n}{A^2} \quad (22)$$

let  $a = (Ak)/n$

Then

$$P(k) = \frac{An^{n-1}}{(n-1)!} a^{n-1} e^{-na}$$

If we replace with  $e = a-1$  then

$$P(k) = \frac{An^n}{n!} (1+e)^{n-1} e^{n(1+e)}$$

Assuming that  $n$  is large, we can express  $P(k)$  in a more convenient manner using Sterling's factorial expansion:<sup>5</sup>

$$n! = \sqrt{(2\pi n)} n e^{-n}$$

Then

$$P(k) = \frac{A}{\sqrt{2\pi n}} (1+e)^{n-1} e^{-ne}$$

But  $e^{-ne} = 1 - \frac{ne}{1!} + \frac{(ne)^2}{2!} \dots$

and  $(1+e)^{n-1} = 1 + (n-1)e + \frac{(n-1)(n-2)e^2}{2} + \dots$

Then

$$P(k) = \frac{A}{\sqrt{2\pi n}} \left(1 - e - \frac{1}{2}(n-2)e^2 + \dots\right)$$

$$= \frac{A}{\sqrt{2\pi n}} e^{\frac{1}{2}n(e-1)^2}$$

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<sup>5</sup>Jeffreys

$$= \frac{A}{\sqrt{2\pi n}} e^{-\frac{1}{2}n(a-1)^2}$$

substituting back for  $a = (Ak)/n$

$$= \frac{A}{\sqrt{2\pi n}} e^{-\frac{1}{2}n(\frac{Ak}{n}-1)^2}$$

$$= \frac{A}{\sqrt{2\pi n}} e^{-\frac{1}{2}\frac{(Ak-n)^2}{n}}$$

$$= \frac{A}{\sqrt{2\pi n}} e^{-\frac{1}{2}\frac{(k-\frac{n}{A})^2}{\frac{n}{A^2}}}$$

But from (19) and (20)  $k_0 = \frac{n}{A}$  and  $\sigma^2 = \frac{n}{A^2}$

Then

$$P(k) = \frac{1}{\sqrt{2\pi\sigma_k}} e^{-\frac{1}{2}\frac{(k-k_0)^2}{(\sigma_k)^2}} \quad (23)$$

Since  $P(k)$  is an expression of  $k$  and since  $k = \frac{2\pi|T|}{i(nd)(rt')}$  then we note that  $P(k^*) = P(-k)$

This provides an expression for the probability of a specific interaction at a specific energy position but it does not account for the spatial location.

To calculate  $P(x)$  the probability of an interaction at a specific spatial position  $x$ , we sum all of the alternative  $P(k)$ 's at any given  $x$  and ensure that we allow for every preceding non-event at  $(x-1), (x-2), \dots$

We know that the probability of NOT having an interaction at spatial position  $x$  is given by  $e^{-ux}$ .

We must consider each  $P(k)$  at a given spatial position  $x$ , over every probability density for each position of  $x$  whilst eliminating every other (NOT  $x$ ) position - along with the sum of every feasible  $P(k)$  at each of these positions:

$$P(x) = \int_{-\infty}^{\infty} P(k) \left\{ \int_{-\infty}^{\infty} P(u-k) \cdot e^{-ux} du \right\} dk \quad (24)$$

Now since  $P^*(k) = P(-k)$  then:

$$P(x) = \int_{-\infty}^{\infty} P(k) \int_{-\infty}^{\infty} P^*(k-u) \cdot e^{-ux} du dk$$

Using the notation for a Fourier Transform where the Fourier Transform of  $P(k)$  is:  $FT(P(k)) = \int_{-\infty}^{\infty} P(k) e^{-ikx} dk$  we can show that:

$$\int_{-\infty}^{\infty} P^*(k-u) e^{-ux} du = \int_{-\infty}^{\infty} P^*(k-u) \cdot e^{-(k-u)x} e^{-ux} d(k-u) = e^{-ux} FT(P^*(k))$$

$$\text{Hence } P(x) = \int_{-\infty}^{\infty} P(k) e^{-kx} FT(P^*(k)) dk$$

$$= FT(P^*(k)) \cdot FT(P(k))$$

Then

$$P(x) = |FT(P(k))|^2 \quad (25)$$

That is the probability of finding an IFE at position  $x$  is the square of the magnitude of the Fourier transform of  $P(k)$ . This is the probabilistic heart of quantum mechanics.

We can consequently define a function  $\psi(x)$  - we will name it “the probability function” - such that  $\psi(x) = FT(P(k))$  and thus:

$$P(x) = |\psi(x)|^2 \quad (26)$$

Since from (21)

$$P(k) = \frac{1}{\sqrt{2\pi}\sigma_k} e^{-\frac{1}{2} \frac{(k-k_0)^2}{(\sigma_k)^2}}$$

Then

$$P(x) = \left| \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_k} e^{-\frac{1}{2} \frac{(k-k_0)^2}{(\sigma_k)^2}} e^{-ikx} dk \right|^2$$

To assist in calculation we can use  $k' = k - k_0$

$$\begin{aligned} P(x) &= \left| \frac{1}{\sqrt{2\pi}\sigma_k} \int_{-\infty}^{\infty} e^{-\frac{\frac{1}{2}(k')^2}{\sigma_k^2}} e^{i(k'+k_0)x} dk' \right|^2 \\ &= \left| \frac{e^{ik_0x}}{\sqrt{2\pi}\sigma_k} \int_{-\infty}^{\infty} e^{-\frac{\frac{1}{2}(k')^2}{\sigma_k^2} + ik'x} dk' \right|^2 \\ &= \left| \frac{e^{ik_0x}}{\sqrt{2\pi}\sigma_k} \int_{-\infty}^{\infty} e^{\frac{1}{2\sigma_k^2}(ik'-\sigma_k^2x)^2} \cdot e^{\frac{1}{2}\sigma_k^2x^2} dk' \right|^2 \end{aligned}$$

Since, from integration tables:  $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\frac{\pi}{x}}$  (where x can be complex):

$$P(x) = \left| \frac{e^{ik_0x}}{\sqrt{2\pi}\sigma_k} e^{\frac{1}{2}\sigma_k^2x^2} \sqrt{\pi 2\sigma_k^2} \right|^2$$

$$i.e. P(x) = 2\pi\sigma_k^2 e^{\sigma_k^2x^2}$$

Thus  $P(x)$  for a free particle also has a Gaussian distribution. A number of significant consequences derive from this explanation.

Although we can derive and make use of the “probability function”  $\psi(x) = FT(P(k))$ , it is a strange creature of mixed real and imaginary heritage - and it lurks like a half human half bull Minotaur in a labyrinth of misunderstanding from which reason never escapes. It has only mathematical significance and no direct reference.

We can, nevertheless, note an interesting feature of the interplay between  $P(k)$  and  $\psi(x) = FT(P(k))$ .

Firstly, we note a property of the differential of  $P(x)$  which we can indicate by  $P'(x)$ :

$$\begin{aligned}\int_{-\infty}^{\infty} P'(x)e^{-ikx}dx &= e^{-ikx}P(x)|_{-\infty}^{\infty} + ik \int_{-\infty}^{\infty} P(x)e^{-ikx}dx \\ \int_{-\infty}^{\infty} P'(x)e^{-ikx}dx &= ikFT(P(k))\end{aligned}\tag{27}$$

Secondly we note that for a combination of such probability functions: say  $P(k)$  and another similar probability function in  $k$   $Q(k)$  then as probability functions are not negative then provided that a factor  $\alpha$  is non-zero (where  $P^*(k)$  indicates the complex conjugate of  $P(k)$ ) :

$$\int_{-\infty}^{\infty} \{P(k) + \alpha Q(k)\}^2 dk > 0$$

Then

$$\int_{-\infty}^{\infty} |P(k)|^2 dk + \alpha^2 \int_{-\infty}^{\infty} |Q(k)|^2 dk + 2\alpha \int_{-\infty}^{\infty} \{P(k)Q(k)\} dk > 0$$

We can solve this as a quadratic equation for  $\alpha$  and hence:

$$\{2 \int_{-\infty}^{\infty} P(k)Q(k)dk\}^2 - 4 \int_{-\infty}^{\infty} |P(k)|^2 dk \int_{-\infty}^{\infty} |Q(k)|^2 dk \leq 0$$

Then

$$\{\int_{-\infty}^{\infty} P(k)Q(k)dk\}^2 \leq \int_{-\infty}^{\infty} |P(k)|^2 dk \int_{-\infty}^{\infty} |Q(k)|^2 dk \quad (28)$$

Using basic definitions for the variance of  $x$  and  $k$ , we form the multiple:

$$\sigma_x^2 \sigma_k^2 = \int_{-\infty}^{\infty} x^2 |P(x)|^2 dx \cdot \int_{-\infty}^{\infty} k^2 |(P(k))|^2 dk$$

However, we can show that:<sup>6</sup>

$$\int_{-\infty}^{\infty} |(P(k))|^2 dk = \int_{-\infty}^{\infty} |FT(P(k))|^2 dk$$

$$\text{Hence } \sigma_x^2 \sigma_k^2 = \int_{-\infty}^{\infty} |xP(x)|^2 dx \cdot \int_{-\infty}^{\infty} |ikFT(P(k))|^2 dk$$

From (25):

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<sup>6</sup>This will be familiar as the proof of the Parseval identity:

$$\begin{aligned} \int_{-\infty}^{\infty} P(k)P^*(k)dk &= \int_{-\infty}^{\infty} P(k)\{\int_{-\infty}^{\infty} FT(P^*(k))e^{rk}\}dk \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(k)e^{rk}P^*(k)dk \\ &= \int_{-\infty}^{\infty} FT(P(k))FT^*(P(k)) \end{aligned}$$

$$= \int_{-\infty}^{\infty} |xP(x)|^2 dx \cdot \int_{-\infty}^{\infty} |P'(x)|^2 dx$$

From (26):

$$\sigma_x^2 \sigma_k^2 \geq \left| \int_{-\infty}^{\infty} x(P(x)P^{*'}(x)) dx \right|^2$$

$$\sigma_x^2 \sigma_k^2 \geq \int_{-\infty}^{\infty} \left| x \frac{d}{dx} |P(x)|^2 \right|^2 dx$$

$$\sigma_x^2 \sigma_k^2 \geq \frac{1}{4} \int_{-\infty}^{\infty} |P(x)|^4 dx$$

And since  $\int_{-\infty}^{\infty} |P(x)|^2 dx = 1$  is the probability of finding the IFE *anywhere*  
 $=1$

Then

$$\sigma_x^2 \sigma_k^2 \geq \frac{1}{4}$$

$$\sigma_x \sigma_k \geq \frac{1}{2} \tag{29}$$

## 7 Momentum and the Concept of Mass

It is useful to consider the apparent combined velocity of an IFE which is moving with a velocity  $v = \frac{d}{\sqrt{(t^*)^2 + (pt')^2}}$  away from a notional fixed reference point and another IFE which is moving away in the other direction from the

fixed reference point at a velocity  $u = \frac{d}{\sqrt{(t^*)^2 + (qt')^2}}$ . This effectively becomes a method for perceiving the resultant velocity of two velocities added together.

We shall consider what occurs in a time  $\sqrt{(t^*)^2 + (pt')^2}$  :

The distance D travelled in this time is:

$$D = d + \frac{d}{\sqrt{(t^*)^2 + (qt')^2}} \sqrt{(t^*)^2 + (pt')^2}$$

However, in analysing the amount of *time* we should employ in formulating the combined velocity of both IFE's, there is a further complication here. During the period of time  $\sqrt{(t^*)^2 + (pt')^2}$  which accounts for a movement in space d for the first IFE, then we have to account for an additional number of *spatial-time* points that would have been covered by the second IFE (determined by the trigger-point qt').

In order to establish how many “extra” incidents of spatial-time  $t^*$  there are in this time, we can consider that in a theoretical amount of time stretching across  $\sqrt{(t^*)^2 + (pt')^2} \sqrt{(t^*)^2 + (qt')^2}$  we will have an extra number N of incidents of  $t^*$  where:

$$N = \sqrt{(\sqrt{(t^*)^2 + (pt')^2} + \sqrt{(t^*)^2 + (qt')^2})^2 - \{(\sqrt{(t^*)^2 + (pt')^2})^2 + (qt')^2\}}$$

This gives us a *rate* of discrepancy of extra  $t^*$  per unit of time such that:

$$rate = \frac{\sqrt{(\sqrt{(t^*)^2 + (pt')^2} + \sqrt{(t^*)^2 + (qt')^2})^2 - \{(\sqrt{(t^*)^2 + (pt')^2})^2 + (qt')^2\}}}{\sqrt{(t^*)^2 + (pt')^2} \sqrt{(t^*)^2 + (qt')^2}}$$

In an amount of time  $\sqrt{(t^*)^2 + (pt')^2}$  there will be  $\frac{\sqrt{(t^*)^2 + (pt')^2}}{\sqrt{(t^*)^2 + (qt')^2}}$  opportunities for an extra “skip” of space-time.

The total number of extra incidents of  $t^*$  will be:

$$\frac{\sqrt{(t^*)^2 + (pt')^2}}{\sqrt{(t^*)^2 + (qt')^2}} \frac{\sqrt{(\sqrt{(t^*)^2 + (pt')^2} + \sqrt{(t^*)^2 + (qt')^2})^2 - \{(\sqrt{(t^*)^2 + (pt')^2})^2 + (qt')^2\}}}{\sqrt{(t^*)^2 + (pt')^2} \sqrt{(t^*)^2 + (qt')^2}}$$

Then the amount of time t we have to consider when calculating the

combined velocity of the two IFEs is:

$$\begin{aligned}
t &= \sqrt{(\sqrt{(t^*)^2 + (pt')^2})^2 + (t^*)^2 \left[ \frac{\sqrt{(t^*)^2 + (pt')^2}}{\sqrt{(t^*)^2 + (qt')^2}} \frac{\sqrt{(\sqrt{(t^*)^2 + (pt')^2} + \sqrt{(t^*)^2 + (qt')^2})^2 - \{(\sqrt{(t^*)^2 + (pt')^2})^2 + (qt')^2\}}}{\sqrt{(t^*)^2 + (pt')^2} \sqrt{(t^*)^2 + (qt')^2}} \right]} \\
&= \sqrt{(\sqrt{(t^*)^2 + (pt')^2})^2 + (t^*)^2 \left[ \frac{2(\sqrt{(t^*)^2 + (pt')^2})}{\sqrt{(t^*)^2 + (qt')^2}} + \frac{(t^*)^2}{(t^*)^2 + (qt')^2} \right]} \\
&= \sqrt{(t^*)^2 + (pt')^2} + \frac{(t^*)^2}{\sqrt{(t^*)^2 + (qt')^2}} + 2(t^*)^2 \frac{\sqrt{(t^*)^2 + (pt')^2}}{\sqrt{(t^*)^2 + (qt')^2}} \\
&= \left[ \sqrt{\sqrt{(t^*)^2 + (pt')^2} + \frac{(t^*)^2}{\sqrt{(t^*)^2 + (qt')^2}}} \right]^2 \\
&= \sqrt{(t^*)^2 + (pt')^2} + \frac{(t^*)^2}{\sqrt{(t^*)^2 + (qt')^2}}
\end{aligned}$$

Then the combined velocity V of the two IFE's is:

$$v = \frac{d + \frac{d}{\sqrt{(t^*)^2 + (qt')^2}} \sqrt{(t^*)^2 + (pt')^2}}{\sqrt{(t^*)^2 + (pt')^2} + \frac{(t^*)^2}{\sqrt{(t^*)^2 + (qt')^2}}} \quad (30)$$

We can now consider a particular case of interest. Consider two IFE's of equal rest mass  $m_0$  and equal velocity  $u$  colliding in a non-elastic way from opposite directions (say a mass moving from the left and a mass moving from the right), resulting in a stationary object of mass  $M_0$ .

We will suppose that mass is not necessarily fixed and hypothesise that the rest mass varies so that the mass  $m(v)$  may be different from the mass  $m_0$  when stationary.

We can also imagine that in this case we sit on the second IFE mass moving from the right. From this perspective the mass moving from the left has an effective combined velocity V and has a mass  $m(V)$ . It then hits the IFE (on which we sit) of mass  $m_0$  which results in an IFE of mass  $m(u)$  moving with a velocity  $u$ .

Effectively velocity V is the combined velocity of two equal velocities each

moving with velocity  $u$ . We can see from equation 30 above that the effective velocity of two combined equal velocities  $u = \frac{d}{\sqrt{(t^*)^2 + (pt')^2}}$  is:

$$V = \frac{2d\sqrt{(t^*)^2 + (pt')^2}}{2(t^*)^2 + (pt')^2} \quad (31)$$

We will now employ two fundamental laws which, I have to say, appear initially as quite grand assumptions. These are:

(1) Conservation of Momentum

i.e.  $m(V)V = m(u)u$

(2) Conservation of Mass

i.e.  $m(V) + m_0 = M(u)$

It will be seen later that these two laws are both concerned with the same fundamental which is the conservation of time.

If we combine these two conservation laws and eliminate  $m(u)$ , we obtain:

$$\frac{mV}{m_0} = \frac{u}{V - u} \quad (32)$$

Making use of  $u = \frac{d}{\sqrt{(t^*)^2 + (pt')^2}}$  and equation 30, we obtain:

$$\begin{aligned} \frac{m(V)}{m_0} &= \frac{\frac{d}{\sqrt{(t^*)^2 + (qt')^2}}}{\frac{2d\sqrt{(t^*)^2 + (qt')^2}}{2(t^*)^2 + (pt')^2} - \frac{d}{\sqrt{(t^*)^2 + (qt')^2}}} = \frac{d}{\frac{2d(t^*)^2 + (qt')^2}{2(t^*)^2 + (pt')^2} - d} \\ \text{Then } \frac{m(V)}{m_0} &= \frac{2(t^*)^2 + (pt')^2}{(pt')^2} \end{aligned} \quad (33)$$

It is instructive to note that if we multiply by  $\frac{d^2}{(t^*)^2}$ :

$$m(V)\frac{d^2}{(t^*)^2} = m_0\frac{d^2}{(t^*)^2} + 2m_0\frac{d^2}{(pt')^2} \quad (34)$$

The second expression on the right indicates a multiple of the rest mass with some form of the square of the velocity.

If we compare with a traditional  $\frac{1}{2}m_0V^2$  Newtonian formulation of kinetic energy, then applying  $V = \frac{2d\sqrt{(t^*)^2+(pt')^2}}{2(t^*)^2+(pt')^2}$ :

$$\begin{aligned} 2m_0\frac{d^2}{(pt')^2} - \frac{1}{2}m_0V^2 &= 2m_0\frac{d^2}{(pt')^2} - \frac{2m_0d^2\{(t^*)^2 + (pt')^2\}}{\{2(t^*)^2 + (pt')^2\}^2} \\ &= \frac{(t^*)^2}{(pt')^2} \frac{2(t^*)^2}{\{2(t^*)^2 + (pt')^2\}^2} \end{aligned}$$

This last term becomes negligible when  $p \gg t^*$  i.e. for speeds much less than the speed of light, and hence for such speeds  $2m_0\frac{d^2}{(pt')^2} \sim \frac{1}{2}m_0V^2$ . This indicates that equation 34 is a relation for the *energy* of the IFE, where the term  $2m_0\frac{d^2}{(pt')^2}$  indicates the kinetic energy of the particle and consequently we have a term for the rest energy of the IFE:

$$E_0 = m_0\frac{d^2}{(t^*)^2}$$

And for the total energy of the IFE:

$$E_T = m(V)\frac{d^2}{(t^*)^2} \tag{35}$$

These are, of course, instances of Einstein's familiar expression  $E = mc^2$ .

We can further note that:

$$m(V)V = m_0\left\{\frac{2(t^*)^2 + (pt')^2}{(pt')^2}\right\}\left\{\frac{2d\sqrt{(t^*)^2 + (pt')^2}}{2(t^*)^2 + (pt')^2}\right\}$$

$$m(V)V = \frac{2dm_0\sqrt{(t^*)^2 + (pt')^2}}{(pt')^2}$$

From (15) we know that  $E = \frac{h}{(rt')}$  and from equation (35)  $m_0 = \frac{h(t^*)^2}{(rt')d^2}$

$$\text{Then } m(V)V = \frac{h2(t^*)^2\sqrt{(t^*)^2 + (pt')^2}}{d(rt')(pt')^2} = \frac{h\sqrt{(t^*)^2 + (pt')^2}}{d(rt')} \frac{2(t^*)^2}{(pt')^2}$$

Yet this is an expression concerning the wavelength of the combined IFE which moves with speed  $u = \frac{d}{\sqrt{(t^*)^2 + (pt')^2}}$  which indicates that:

$$m(V)V = \frac{h}{\lambda} \quad (36)$$

Since  $p = \frac{h}{\lambda} = \frac{hk}{2\pi}$  then

$$\sigma_x \sigma_p \geq \frac{h}{4\pi} \quad (37)$$

This is the familiar expression of Heisenberg's Uncertainty Principle.

## 8 Conclusions

The explanations covered in this paper underpin the theories of relativity and quantum physics - and provide clarifying reasons for some confusing aspects of these theories - including why the speed of light has a maximum, why mass should be connected to energy, the perceived differences in experienced time for moving and stationary entities, how the concepts for the speed of light

$c$  and Planck's constant relate more fundamentally to the units of space-time  $t^*$  and energy-time  $t'$ , and the quantum uncertainty of position and momentum. A later paper is intended on the theory of gravity.

## 9 References

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